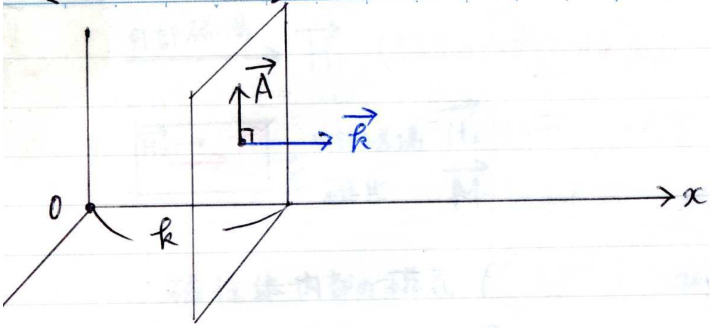
 九高理の後 勉強開始

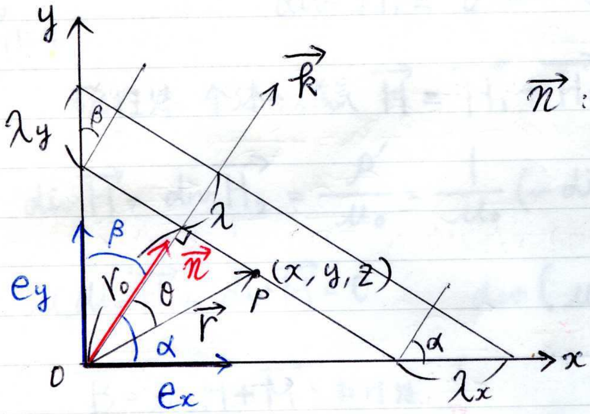
[電磁波の表し方]



x軸方向に進む平面波の一般式

$$\vec{A} = \vec{A_0}\sin(\omega t - kx) \quad —①$$

$\left(k=\frac{2\pi}{\lambda}\text{とする}\right)$



$\vec{n}$：平面波の進行方向の法線単位ベクトル

$$\vec{n} = (\cos\alpha, \cos\beta, \cos\gamma)$$

$\lambda_x \cos\alpha = \lambda$ より $\quad \lambda_x = \dfrac{\lambda}{\cos\alpha}$ —②

同様に

$$\lambda_y = \frac{\lambda}{\cos\beta} \quad \lambda_z = \frac{\lambda}{\cos\gamma} \quad —③$$

波数 $k = \dfrac{2\pi}{\lambda}$ より

$$k_x = \frac{2\pi}{\lambda_x} = \frac{2\pi}{\lambda}\cos\alpha = k\cos\alpha$$

$$k_y = \frac{2\pi}{\lambda_y} = \frac{2\pi}{\lambda}\cos\beta = k\cos\beta$$

$$k_z = \frac{2\pi}{\lambda_z} = \frac{2\pi}{\lambda}\cos\gamma = k\cos\gamma$$

—④

単位ベクトルを $\vec{e_x}$, $\vec{e_y}$, $\vec{e_z}$ とおくと

$$\vec{k} = k_x \vec{e_x} + k_y \vec{e_y} + k_z \vec{e_z}$$ と定義すれば

$$= k\cos\alpha\,\vec{e_x} + k\cos\beta\,\vec{e_y} + k\cos\gamma\,\vec{e_z}$$

$$\vec{k} = k\{\cos\alpha\,\vec{e_x} + \cos\beta\,\vec{e_y} + \cos\gamma\,\vec{e_z}\} = k\cdot\vec{n} \quad ⑤$$

$\vec{k}$ は 波数とその伝播方向を示すベクトル → $\boxed{\text{波数ベクトル}}$

平面波 $\vec{A}$ を角周波数 ω の正弦波とすると，原点 O から 波面までの時刻 t における垂直距離を r_0 とすれば

$$\vec{A} = \vec{A_0}\sin(\omega t - k r_0)$$ と書ける ⑥　　$\vec{A_0}$ は振幅

この位相中の $k r_0$

平面波の一点 P の 原点 O からの位置ベクトルを $\vec{OP} = \vec{r}$ とすれば

$P(x, y, z)$

$$k r_0 = k r\cos\theta = \vec{k}\cdot\vec{r} = \underbrace{k\cdot\vec{n}}_{\vec{k}}\cdot\vec{r} = k_x x + k_y y + k_z z \quad ⑦$$

θ は $\vec{k}$ と $\vec{r}$ のなす角

$$\therefore\ \vec{A} = \vec{A_0}\sin(\omega t - k r_0) = \vec{A_0}\sin(\omega t - \vec{k}\cdot\vec{r}) \quad ⑧$$

伝播速度 $v = \dfrac{\lambda}{T} = \dfrac{\frac{\lambda}{2\pi}}{\frac{T}{2\pi}} = \dfrac{\omega}{k}$

$$\vec{v} = \frac{\omega}{k}\vec{n} = \frac{\omega}{k}\frac{\vec{k}}{k} = \frac{\omega}{k^2}\vec{k} \quad ⑨$$

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Maxwell equation より ①〜④

$$\begin{cases} \mathrm{div}\,\vec{D} = \rho \quad (\text{真電荷}) & —① \\ \mathrm{div}\,\vec{B} = 0 & —② \\ \mathrm{rot}\,\vec{H} = \vec{j} + \dfrac{\partial \vec{D}}{\partial t} & —③ \\ \mathrm{rot}\,\vec{E} = -\dfrac{\partial \vec{B}}{\partial t} & —④ \end{cases} \quad \text{また} \quad \begin{cases} \vec{D} = \varepsilon \vec{E} & —⑤ \\ \vec{B} = \mu \vec{H} & —⑥ \\ \vec{j} = \sigma \vec{E} & —⑦ \end{cases}$$

簡単の為に 空間において $\rho = 0$ $\vec{j} = 0$ $\varepsilon = $ 一定 $\mu = $ 一定

①〜④ は

$$\mathrm{div}\,\vec{D} = 0 \quad —⑧ \qquad \varepsilon\,\mathrm{div}\,\vec{E} = 0 \qquad \mathrm{div}\,\vec{E} = 0 \quad —⑧'$$

$$\mathrm{div}\,\vec{B} = 0 \quad —⑨ \longrightarrow \mu\,\mathrm{div}\,\vec{H} = 0 \qquad \mathrm{div}\,\vec{H} = 0 \quad —⑨'$$

$$\mathrm{rot}\,\vec{H} = \frac{\partial \vec{D}}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t} \quad —⑩$$

$$\mathrm{rot}\,\vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad —⑪$$

⑩を t で微分する

$$\frac{\partial}{\partial t}\mathrm{rot}\,\vec{H} = \mathrm{rot}\frac{\partial \vec{H}}{\partial t}$$

⑩式は $\longrightarrow$ $\mathrm{rot}\dfrac{\partial \vec{H}}{\partial t} = \varepsilon \dfrac{\partial^2 \vec{E}}{\partial t^2}$ —⑫ となる

⑪ の 両辺の rot をとると

$$\mathrm{rot}\,\mathrm{rot}\,\vec{E} = -\mu\,\mathrm{rot}\frac{\partial \vec{H}}{\partial t}$$

$$\mathrm{grad}(\underbrace{\mathrm{div}\,\vec{E}}_{0}) - \nabla^2 \vec{E} = -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\therefore \quad -\nabla^2 \vec{E} = -\varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} \quad —⑬$$

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⑪の両辺をtで微分する

$$\frac{\partial}{\partial t}\mathrm{rot}\vec{E} = -\mu\frac{\partial^2\vec{H}}{\partial t^2}$$

$$\mathrm{rot}\frac{\partial\vec{E}}{\partial t} = -\mu\frac{\partial^2\vec{H}}{\partial t^2} \quad — ⑭$$

⑪の両辺のrotをとると

$$\mathrm{rot}\,\mathrm{rot}\vec{H} = \mathrm{rot}\left(\varepsilon\frac{\partial\vec{E}}{\partial t}\right) = \varepsilon\,\mathrm{rot}\frac{\partial\vec{E}}{\partial t} = -\varepsilon\mu\frac{\partial^2\vec{H}}{\partial t^2} \quad — ⑮$$

$$\mathrm{grad}(\underbrace{\mathrm{div}\vec{H}}_{0}) - \nabla^2\vec{H} = -\varepsilon\mu\frac{\partial^2\vec{H}}{\partial t^2} \quad — ⑯$$

$$-\nabla^2\vec{H} = -\varepsilon\mu\frac{\partial^2\vec{H}}{\partial t^2} \quad — ⑰$$

$$\nabla^2\vec{H} = \varepsilon\mu\frac{\partial^2\vec{H}}{\partial t^2} \quad — ⑱$$

⑬ ⑱ は波動方程式

⑬, ⑱ の解を

$$\left.\begin{aligned}\vec{E} &= \vec{E_0}\sin(\omega t - \vec{k}\cdot\vec{r}) \quad —⑲\\ \vec{H} &= \vec{H_0}\sin(\omega t - \vec{k}\cdot\vec{r}) \quad —⑳\end{aligned}\right\}$$

の平面波として $\vec{E_0}, \vec{H_0}$ の性質を調べる。

$$\mathrm{div}\vec{E} = \mathrm{div}\vec{E_0}\cdot\sin(\omega t - \vec{k}\cdot\vec{r}) + \vec{E_0}\cdot\nabla(\sin(\omega t - \vec{k}\cdot\vec{r}))$$

$$= (E_{0x}\vec{e_x} + E_{0y}\vec{e_y} + E_{0z}\vec{e_z})\cdot(-k_x\vec{e_x} - k_y\vec{e_y} - k_z\vec{e_z})\cdot\cos(\ )$$

$$= -(+k_xE_{0x} + k_yE_{0y} + k_zE_{0z})\cos(\omega t - \vec{k}\cdot\vec{r}) = 0$$

$$\therefore -\vec{k}\cdot\vec{E_0}\cos(\omega t - \vec{k}\cdot\vec{r}) = 0$$

$$\therefore \vec{k}\cdot\vec{E_0} = 0 \quad —㉑ \qquad \left\{\begin{aligned}&\vec{k}\perp\vec{E_0}\\ &\vec{k}\perp\vec{H_0}\end{aligned}\right.$$

同様に $\mathrm{div}\vec{H} = 0$ から

$$\vec{k}\cdot\vec{H_0} = 0 \quad —㉒$$

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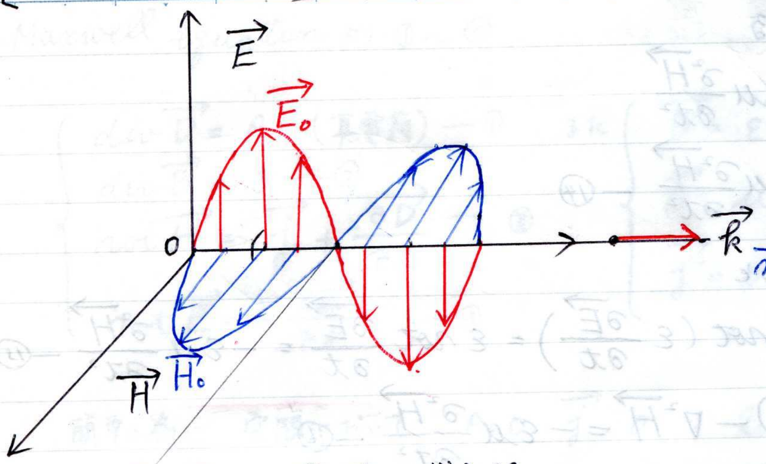
[電場と磁場の直交性]



⑲, ⑳ と ⑩, ⑪ に代入する

$$rot\left(\vec{H_0}\cdot\sin(\omega t-\vec{k}\cdot\vec{r})\right)=\varepsilon\frac{\partial}{\partial t}\left(\vec{E_0}\sin(\omega t-\vec{k}\cdot\vec{r})\right) \quad ㉓$$

$$rot\left\{\sin(\omega t-\vec{k}\cdot\vec{r})\cdot\vec{H_0}\right\}=\varepsilon\frac{\partial}{\partial t}\left\{\vec{E_0}\sin(\omega t-\vec{k}\cdot\vec{r})\right\} \quad ㉔$$

$$\nabla\{\sin(\omega t-\vec{k}\cdot\vec{r})\}\times\vec{H_0}+\sin(\omega t-\vec{k}\cdot\vec{r})\cdot rot\vec{H_0}$$

$$-\vec{k}\cos(\omega t-\vec{k}\cdot\vec{r})\times\vec{H_0}=\varepsilon\left\{\vec{E_0}\omega\cos(\omega t-\vec{k}\cdot\vec{r})\right\}$$

$$\vec{H_0}\times\vec{k}\,\cancel{\cos(\omega t-\vec{k}\cdot\vec{r})}=\omega\varepsilon\vec{E_0}\,\cancel{\cos(\omega t-\vec{k}\cdot\vec{r})}$$

$$\therefore\ \vec{H_0}\times\vec{k}=\omega\varepsilon\vec{E_0} \quad ㉕$$

$$rot\left\{\vec{E_0}\sin(\omega t-\vec{k}\cdot\vec{r})\right\}=-\mu\frac{\partial}{\partial t}\left\{\vec{H_0}\sin(\omega t-\vec{k}\cdot\vec{r})\right\}$$

㉔と同様に

$$rot\left\{\sin(\omega t-\vec{k}\cdot\vec{r})\cdot\vec{E_0}\right\}=-\mu\frac{\partial}{\partial t}\left\{\vec{H_0}\sin(\omega t-\vec{k}\cdot\vec{r})\right\}$$

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$$\vec{E_0} \times \vec{k} = -\omega\mu \vec{H_0}$$

$$-\vec{k} \times \vec{E_0} = -\omega\mu \vec{H_0} \qquad \therefore \vec{k} \times \vec{E_0} = \omega\mu \vec{H_0} \quad —㉖$$

$\therefore$ $\vec{E_0}$, $\vec{H_0}$, $\vec{k}$ は右手系をなす.

$$v = \frac{\omega}{k} \qquad v = \frac{1}{\sqrt{\varepsilon\mu}} \text{ より}$$

$$\frac{1}{\sqrt{\varepsilon\mu}} = \frac{\omega}{k} \qquad \omega = \frac{k}{\sqrt{\varepsilon\mu}} \quad —㉗$$

㉗を㉕に代入すると

$$\vec{H_0} \times \vec{k} = \frac{k\varepsilon}{\sqrt{\varepsilon\mu}} \vec{E_0} = k\sqrt{\frac{\varepsilon}{\mu}} \vec{E_0} \quad —、$$

$$\vec{H_0} \times \underbrace{\frac{\vec{k}}{k}}_{n} = \sqrt{\frac{\varepsilon}{\mu}} \vec{E_0} \quad —㉘$$

㉖に代入

$$\vec{k} \times \vec{E_0} = \frac{k\mu}{\sqrt{\varepsilon\mu}} \vec{H_0} = k\sqrt{\frac{\mu}{\varepsilon}} \vec{H_0}$$

$$\underbrace{\frac{\vec{k}}{k}}_{n} \times \vec{E_0} = \sqrt{\frac{\mu}{\varepsilon}} \vec{H_0} \quad —㉙$$

$$\therefore \quad |E_0| = \sqrt{\frac{\mu}{\varepsilon}} |H_0| \quad \therefore \frac{|E_0|}{|H_0|} = \sqrt{\frac{\mu}{\varepsilon}} = Z \quad —㉚$$

㉖より

$$\vec{E_0} \cdot (\vec{k} \times \vec{E_0}) = \omega\mu \vec{E_0} \cdot \vec{H_0}$$

$\therefore$

$$\vec{k} \cdot (\underbrace{\vec{E_0} \times \vec{E_0}}_{0}) = \omega\mu \vec{E_0} \cdot \vec{H_0}$$

$$\therefore \quad \vec{E_0} \cdot \vec{H_0} = 0 \quad —㉛$$