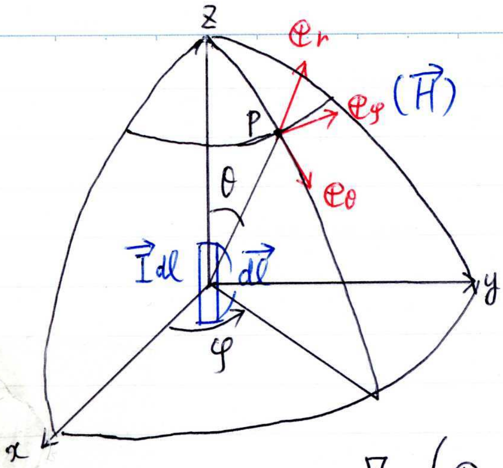
Date 20 8 18

(電磁波の放射)



P点のベクトルポテンシャル

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{I(t-\frac{r}{c})\vec{dl}}{r} \quad \text{①}$$

$$\vec{B} = \mathrm{rot}\,\vec{A} = \nabla \times \vec{A} \quad \text{②}$$

$$\nabla = \left(e_r\frac{\partial}{\partial r} + e_\theta\frac{\partial}{r\partial\theta} + e_\varphi\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\right) \quad \text{③}$$

②に代入すると

$$\left(e_r\frac{\partial}{\partial r} + e_\theta\frac{\partial}{r\partial\theta} + e_\varphi\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\right)\times\frac{\mu_0}{4\pi}\frac{I(t-\frac{r}{c})}{r}\vec{dl} \quad \text{④}$$

$$= e_r\times\frac{\partial}{\partial r}\left\{\left(\frac{\mu_0}{4\pi}\frac{I(t-\frac{r}{c})}{r}\right)\cdot\vec{dl}\right\} + e_\theta\times\frac{\partial}{r\partial\theta}\left\{\left(\frac{\mu_0}{4\pi}\frac{I(t-\frac{r}{c})}{r}\right)\cdot\vec{dl}\right\}$$

$$+ e_\varphi\times\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left\{\left(\frac{\mu_0}{4\pi}\frac{I(t-\frac{r}{c})}{r}\right)\cdot\vec{dl}\right\} \quad \text{⑤}$$

$$= e_r\times\frac{\partial}{\partial r}\left[\frac{\mu_0}{4\pi}\frac{I(t-\frac{r}{c})}{r}\right]\cdot\vec{dl} \quad \text{⑥}$$

$$\vec{B} = \frac{\partial}{\partial r}\left[\frac{\mu_0}{4\pi}\frac{I(t-\frac{r}{c})}{r}\right]e_r\times\vec{dl} \quad \text{⑦}$$

$$\vec{dl}\times e_r = dl\sin\theta\, e_\varphi$$

$$= \frac{\partial}{\partial r}\left[\frac{\mu_0}{4\pi}\frac{I(t-\frac{r}{c})}{r}\right](-dl\sin\theta\, e_\varphi)$$

$$= -\frac{\partial}{\partial r}\left[\frac{\mu_0}{4\pi}\frac{I(t-\frac{r}{c})}{r}\right]\sin\theta\, dl\cdot e_\varphi$$

r^{-1}

$$-\frac{\mu_0}{4\pi}\frac{\partial}{\partial r}\left[\frac{I(t-\frac{r}{c})}{r}\right]\sin\theta\cdot dl\, e_\varphi$$

$$=-\frac{\mu_0}{4\pi}\left[-\frac{1}{r^2}I(t-\frac{r}{c})+\frac{1}{r}\frac{\partial I(t-\frac{r}{c})}{\partial r}\right]\sin\theta\, dl\cdot e_\varphi$$

$T=t-\frac{r}{c}$

$$\frac{\partial I(t-\frac{r}{c})}{\partial r}=\frac{\partial I}{\partial t}\cdot\left(-\frac{1}{c}\right) \qquad \frac{\partial}{\partial T}=\frac{\partial}{\partial t}$$

$$=-\frac{\partial I}{\partial t}\frac{1}{c}$$

$$\frac{\partial}{\partial r}I(t-\frac{r}{c})=-\frac{1}{c}\frac{\partial I}{\partial t} \qquad -\frac{\partial}{\partial r}\left(\frac{cI(t-\frac{r}{c})}{\partial r}\right.$$

$$=-\frac{\mu_0}{4\pi}\left[-\frac{1}{r^2}I(t-\frac{r}{c})-\frac{1}{rc}\frac{\partial}{\partial t}I(t-\frac{r}{c})\right]\sin\theta\, dl\cdot e_\varphi \quad \text{—⑧}$$

$$\vec{B}=\frac{\mu_0}{4\pi}\left\{\frac{I(t-\frac{r}{c})}{r^2}+\frac{1}{rc}\frac{\partial}{\partial t}I(t-\frac{r}{c})\right\}\sin\theta\cdot dl\, e_\varphi \quad \text{—⑨}$$

電界 $\vec{E}$

ローレンツの条件より

$$\mathrm{div}\vec{A}+\frac{1}{c^2}\frac{\partial\phi}{\partial t}=0 \quad \text{—①}$$

$$\frac{\partial\phi}{\partial t}=-\frac{1}{c^2}\mathrm{div}\vec{A} \quad \text{—②}$$

$$\phi=\int\frac{\partial\phi}{\partial t}dt=-\frac{1}{c^2}\int\mathrm{div}\vec{A}\,dt=-\frac{1}{c^2}\int\mathrm{div}\left(\frac{\mu_0}{4\pi}\frac{I(t-\frac{r}{c})}{r}\vec{dl}\right)dt \quad \text{—③}$$

$$=-\frac{1}{c^2}\int\left(e_r\frac{\partial}{\partial r}+e_\theta\frac{\partial}{r\partial\theta}+e_\varphi\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\right)\left\{\frac{\mu_0}{4\pi}\frac{I(t-\frac{r}{c})}{r}\cdot\vec{dl}\right\}dt \quad \text{—④}$$

$$=-\frac{1}{c^2}\int e_r\frac{\partial}{\partial r}\left[\frac{\mu_0}{4\pi}\frac{I(t-\frac{r}{c})}{r}\vec{dl}\right\}dt \quad \text{—⑤}$$

$$=-\frac{1}{c^2}\int\frac{\partial}{\partial r}\left[\frac{\mu_0}{4\pi}\frac{I(t-\frac{r}{c})}{r}\right]e_r\cdot\vec{dl}\,dt=-\frac{\mu_0}{\varepsilon_0\mu_0 4\pi}\int\frac{\partial}{\partial r}\left[\frac{I(t-\frac{c}{r})}{r}\right]\cos\theta\, dl\, dt \quad \text{—⑥}$$

$$= -\frac{1}{4\pi\varepsilon_0}\int \frac{\partial}{\partial r}\left[\frac{I(t-\frac{r}{c})}{r}\right]\cos\theta\, dl\cdot dt$$

$$\frac{\partial}{\partial r}\left[\frac{I(t-\frac{r}{c})}{r}\right] = \left\{-\frac{1}{r^2}I(t-\frac{r}{c})+\frac{\partial I}{r\partial t}\left(-\frac{1}{c}\right)\right\}$$

$$= -\left\{\frac{I}{r^2}+\frac{1}{rc}\frac{\partial I}{\partial t}\right\}$$

$$= +\frac{1}{4\pi\varepsilon_0}\int\left\{\frac{I}{r^2}+\frac{1}{rc}\frac{\partial I}{\partial t}\right\}\cos\theta\, dl\, dt \quad —⑦$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\int\cos\theta\, dl\left\{\frac{I(t-\frac{r}{c})}{r^2}+\frac{1}{rc}\frac{\partial I(t-\frac{r}{c})}{\partial t}\right\}dt \quad —⑧$$

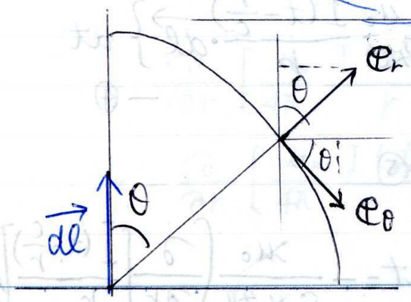
$\vec{E} = -\,\mathrm{grad}\,\phi - \frac{\partial \vec{A}}{\partial t}$ より —⑨

$$= -\left(e_r\frac{\partial}{\partial r}+e_\theta\frac{\partial}{r\partial\theta}+e_\varphi\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\right)\phi - \frac{\partial}{\partial t}\left\{\frac{\mu_0}{4\pi}\frac{I(t-\frac{r}{c})}{r}\vec{dl}\right.$$

$$= -\left(e_r\frac{\partial}{\partial r}+e_\theta\frac{\partial}{r\partial\theta}\right)\frac{1}{4\pi\varepsilon_0}\int\cos\theta\, dl\left(\frac{I}{r^2}+\frac{1}{rc}\frac{\partial I}{\partial t}\right)dt +$$

$$= -\frac{dl}{4\pi\varepsilon_0}\int\left(e_r\frac{\partial}{\partial r}+e_\theta\frac{\partial}{r\partial\theta}\right)\left\{\cos\theta\left(\frac{I}{r^2}+\frac{1}{rc}\frac{\partial I}{\partial t}\right)\right\}dt +$$

$$-\frac{\mu_0}{4\pi}\cdot\frac{\partial}{\partial t}\left\{\frac{I\cdot\vec{dl}}{r}\right\} \quad —⑪$$



$$\vec{dl} = dl(\cos\theta\, e_r - \sin\theta\, e_\theta)$$

$$-\frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\left\{I\cdot\frac{dl}{r}(\cos\theta\, e_r - \sin\theta\, e_\theta)\right\}$$

$$\vec{E} = -\frac{dl}{4\pi\varepsilon_0}\int \mathbb{e}_r \frac{\partial}{\partial r}\left\{\cos\theta\left(\frac{I}{r^2}+\frac{1}{rc}\frac{\partial I}{\partial t}\right)\right\}dt$$

$$-\frac{dl}{4\pi\varepsilon_0}\int \mathbb{e}_\theta \frac{\partial}{r\partial\theta}\left\{\cos\theta\left(\frac{I}{r^2}+\frac{1}{rc}\frac{\partial I}{\partial t}\right)\right\}dt$$

$$-\frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\left\{I\frac{dl}{r}\cos\theta\,\mathbb{e}_r\right\}+\frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\left\{I\frac{dl}{r}\sin\theta\,\mathbb{e}_\theta\right\} \quad \text{—⑫}$$

$$\vec{E} = -\frac{dl}{4\pi\varepsilon_0}\mathbb{e}_r\cdot\cos\theta\int\frac{\partial}{\partial r}\left(\frac{I}{r^2}+\frac{1}{rc}\frac{\partial I}{\partial t}\right)dt$$

$$-\frac{dl}{4\pi\varepsilon_0}\mathbb{e}_\theta\int\frac{\partial}{r\partial\theta}\left\{\cos\theta\left(\frac{I}{r^2}+\frac{1}{rc}\frac{\partial I}{\partial t}\right)\right\}dt$$

$$-\frac{\mu_0}{4\pi}\cos\theta\frac{dl}{r}\frac{\partial I}{\partial t}\mathbb{e}_r+\frac{\mu_0}{4\pi}\frac{dl}{r}\sin\theta\,\mathbb{e}_\theta\frac{\partial I}{\partial t}$$

$$= -\frac{dl\cos\theta}{4\pi\varepsilon_0}\mathbb{e}_r\left[\int\frac{\partial}{\partial r}\left(\frac{I}{r^2}+\frac{1}{rc}\frac{\partial I}{\partial t}\right)dt+\mu_0\varepsilon_0\frac{1}{r}\frac{\partial I}{\partial t}\right]$$ → P6へ

⑩

$$-\frac{dl}{4\pi\varepsilon_0}\mathbb{e}_\theta\left[\int\frac{\partial}{r\partial\theta}\left\{\cos\theta\left(\frac{I}{r^2}+\frac{1}{rc}\frac{\partial I}{\partial t}\right)\right\}dt-\mu_0\varepsilon_0\frac{\sin\theta}{r}\frac{\partial I}{\partial t}\right]$$ P6へ

$$\left[\int\frac{-1}{r}\left\{\sin\theta\left(\frac{I}{r^2}+\frac{1}{rc}\frac{\partial I}{\partial t}\right)\right\}dt-\frac{\mu_0\varepsilon_0\sin\theta}{r}\frac{\partial I}{\partial t}\right]$$

$$-\sin\theta\left[\int\left\{\frac{I}{r^3}+\frac{1}{r^2c}\frac{\partial I}{\partial t}\right\}dt+\frac{\mu_0\varepsilon_0}{r}\frac{\partial I}{\partial t}\right]$$

$$\cancel{-2 \frac{I}{r^3} - \frac{2}{r^2 c}\frac{\partial I}{\partial t} - \frac{1}{r^2 c}\frac{\partial^2 I}{\partial t^2}}$$

$$\cancel{-2\frac{I}{r^3} -}$$

$$\int\left[\frac{\partial}{\partial r}\left(\frac{I}{r^2}\right)dt + \int\left\{\frac{\partial}{\partial r}\left(\frac{1}{cr}\frac{\partial I}{\partial t}\right)\right\}dt\right]$$

$$\int \cos\theta\left\{-\frac{2}{r^3}I + \frac{1}{r^2}\frac{\partial I}{\partial r} + \left(-\frac{1}{cr^2}\right)\frac{\partial I}{\partial t} + \frac{1}{cr}\frac{\partial}{\partial r}\left(\frac{\partial I}{\partial t}\right)\right\}dt$$

$$\frac{1}{r^2}\left(-\frac{\partial I}{\partial t}\frac{1}{c}\right) + \frac{1}{cr}\left(-\frac{\partial}{c\partial t}\left(\frac{\partial I}{\partial t}\right)\right\}$$

$$\int\left\{-\frac{2}{r^3}I - \frac{2}{cr^2}\frac{\partial I}{\partial t} - \frac{1}{c^2 r}\frac{\partial}{\partial t}\left(\frac{\partial I}{\partial t}\right)\right\}dt \qquad \mu_0\varepsilon_0 = \frac{1}{c^2}$$

$$\frac{1}{c^2 r}\frac{\partial^2 I}{\partial t^2}$$

$$= \left[-2\int\left\{\frac{I}{r^3}\right\}dt - \frac{2}{cr^2}I - \cancel{\frac{1}{c^2 r}\frac{\partial I}{\partial t}} + \cancel{\frac{1}{c^2 r}\frac{\partial I}{\partial t}}\right]$$

$$= -2\left[\frac{1}{r^3}\int I\,dt + \frac{I}{cr^2}\right]$$

$$-\frac{dl\cos\theta}{4\pi\varepsilon_0}\times(-2)\left[\frac{1}{r^3}\int I\,dt + \frac{I}{cr^2}\right]\vec{e_r}$$

$$-\frac{dl}{4\pi\varepsilon_0}(-\sin\theta)\left[\int\left\{\frac{I}{r^3} + \frac{1}{r^2 c}\frac{\partial I}{\partial t}\right\}dt + \frac{\mu_0\varepsilon_0}{r}\frac{\partial I}{\partial t}\right]\cdot\vec{e_\theta}$$

$$\vec{E} = \frac{dl\cos\theta}{2\pi\varepsilon_0}\left[\frac{1}{r^3}\int I\,dt + \frac{I}{cr^2}\right]\vec{e_r}$$

$$+ \frac{dl\sin\theta}{4\pi\varepsilon_0}\left[\frac{1}{r^3}\int I\,dt + \frac{1}{r^2c}I + \frac{1}{c^2r}\frac{\partial I}{\partial t}\right]\vec{e_\theta}$$

$$\boxed{\vec{E} = \frac{dl}{2\pi\varepsilon_0}\left[\frac{1}{r^3}\int I\left(t-\frac{r}{c}\right)dt + \frac{I\left(t-\frac{c}{r}\right)}{cr^2}\right]\cos\theta\,\vec{e_r} + \frac{dl}{4\pi\varepsilon_0}\left[\frac{1}{r^3}\int I\left(t-\frac{r}{c}\right)dt + \frac{I\left(t-\frac{c}{r}\right)}{r^2c} + \frac{1}{c^2r}\frac{\partial I\left(t-\frac{c}{r}\right)}{\partial t}\right]\sin\theta\,\vec{e_\theta}}$$

$$\boxed{\vec{H} = \frac{1}{4\pi}\left\{\frac{I\left(t-\frac{r}{c}\right)}{r^2} + \frac{1}{rc}\frac{\partial}{\partial t}I\left(t-\frac{r}{c}\right)\right\}\sin\theta\,dl\,\vec{e_\varphi}}$$

$\frac{1}{r^3}$	で減衰する	$\longrightarrow$	静電磁界
$\frac{1}{r^2}$	〃	$\longrightarrow$	誘導電磁界
$\frac{1}{r}$	〃	$\longrightarrow$	放射電磁界

→ 遠方では，この項だけが残ると考えてよい。

$I = I_0 e^{i\omega t}$ で変化するとき

$$\frac{\partial I}{\partial t} = I_0 i\omega e^{i\omega t} = iI_0\omega e^{i\omega t}$$