［ディラック方程式の確率密度の流れ]

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \qquad \psi^\dagger = \left(\psi^*\right)^T = \begin{pmatrix} \psi_1{}^* & \psi_2{}^* & \psi_3{}^* & \psi_4{}^* \end{pmatrix} \cdots ①$$

$$\overline{\psi} = \psi^\dagger \gamma^0 = \begin{pmatrix} \psi_1{}^* & \psi_2{}^* & \psi_3{}^* & \psi_4{}^* \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} \psi_1{}^* & \psi_2{}^* & -\psi_3{}^* & -\psi_4{}^* \end{pmatrix} \cdots ②$$

$$\left(i\hbar\gamma^\mu\partial_\mu - m\right)\psi = 0 \cdots ③ \rightarrow \quad \left(i\hbar\gamma^\mu\partial_\mu\psi\right)^\dagger - \left(m\psi\right)^\dagger = 0 \quad \rightarrow \quad -i\hbar\partial_\mu\psi^\dagger\left(\gamma^\mu\right)^\dagger\gamma^0 - m\psi^\dagger\gamma^0 = 0$$

$$i\hbar\partial_\mu\psi^\dagger\gamma^\mu\gamma^0 - m\psi^\dagger\gamma^0 = 0 \quad \rightarrow \quad -i\hbar\partial_\mu\psi^\dagger\gamma^0\gamma^\mu - m\psi^\dagger\gamma^0 = 0 \rightarrow i\hbar\partial_\mu\psi^\dagger\gamma^0\gamma^\mu + m\psi^\dagger\gamma^0 = 0$$

$$\therefore \quad i\hbar\partial_\mu\overline{\psi}\gamma^\mu + m\overline{\psi} = 0 \cdots ④$$

$$i\hbar\partial_\mu\overline{\psi}\gamma^\mu\psi + m\overline{\psi}\psi = 0 \quad \cdots ⑤ \quad i\hbar\partial_\mu\left(\overline{\psi}\gamma^\mu\psi\right) = i\hbar\overline{\psi}\gamma^\mu\partial_\mu\psi + i\hbar\left(\partial_\mu\overline{\psi}\right)\gamma^\mu\psi = m\overline{\psi}\psi - m\overline{\psi}\psi = 0 \cdots ⑥$$

$\partial_\mu\left(\overline{\psi}\gamma^\mu\psi\right) = 0 \quad \cdots ⑦ \qquad J^\mu = \overline{\psi}\gamma^\mu\psi \quad \cdots ⑧$ と確率密度をおけば

$$\partial_\mu J^\mu = 0 \cdots ⑨ \qquad J^\mu = \begin{pmatrix} \overline{\psi}\gamma^0\psi & \overline{\psi}\vec{\gamma}\psi \end{pmatrix}$$

$\overline{\psi}\gamma^0\psi = \psi^\dagger\gamma^0\gamma^0\psi = \psi^\dagger\left(\gamma^0\right)^2\psi = \psi^\dagger\psi = \rho$ とおくと $J^\mu = \begin{pmatrix} \rho & \vec{j} \end{pmatrix}$ となる。