［Dirac equation の導出]

$$\left(\gamma^k p_k - mc\right)\left(\gamma^\lambda p_\lambda + mc\right)$$

$$= \left(\gamma^0 p_0 + \gamma^1 p_1 + \gamma^2 p_2 + \gamma^3 p_3 - mc\right)\left(\gamma^0 p_0 + \gamma^1 p_1 + \gamma^2 p_2 + \gamma^3 p_3 + mc\right)$$

$$= \left(\gamma^0 p^0 - \gamma^1 p^1 - \gamma^2 p^2 - \gamma^3 p^3 - mc\right)\left(\gamma^0 p^0 - \gamma^1 p^1 - \gamma^2 p^2 - \gamma^3 p^3 + mc\right)$$

$$\begin{aligned}
&= \left(\gamma^0\right)^2\left(p^0\right)^2 + \left(\gamma^1\right)^2\left(p^1\right)^2 + \left(\gamma^2\right)^2\left(p^2\right)^2 + \left(\gamma^3\right)^2\left(p^3\right)^2 - (mc)^2 \\
&\quad - \gamma^0\gamma^1 p^0 p^1 - \gamma^0\gamma^2 p^0 p^2 - \gamma^0\gamma^3 p^0 p^3 \\
&\quad - \gamma^1\gamma^0 p^1 p^0 + \gamma^1\gamma^2 p^1 p^2 + \gamma^1\gamma^3 p^1 p^3 \\
&\quad - \gamma^2\gamma^0 p^2 p^0 + \gamma^2\gamma^1 p^2 p^1 + \gamma^2\gamma^3 p^2 p^3 \\
&\quad - \gamma^3\gamma^0 p^3 p^0 + \gamma^3\gamma^1 p^3 p^1 + \gamma^3\gamma^2 p^3 p^2
\end{aligned}$$

$$\begin{aligned}
&= \left(\gamma^0\right)^2\left(p^0\right)^2 + \left(\gamma^1\right)^2\left(p^1\right)^2 + \left(\gamma^2\right)^2\left(p^2\right)^2 + \left(\gamma^3\right)^2\left(p^3\right)^2 - (mc)^2 \\
&- \left(\gamma^0\gamma^1 + \gamma^1\gamma^0\right)p^0 p^1 - \left(\gamma^0\gamma^2 + \gamma^2\gamma^0\right)p^0 p^2 - \left(\gamma^0\gamma^3 + \gamma^3\gamma^0\right)p^0 p^3 \\
&+ \left(\gamma^1\gamma^2 + \gamma^2\gamma^1\right)p^1 p^2 + \left(\gamma^1\gamma^3 + \gamma^3\gamma^1\right)p^1 p^3 + \left(\gamma^2\gamma^3 + \gamma^3\gamma^2\right)p^2 p^3
\end{aligned}$$

$$\left(\gamma^0\right)^2 = 1$$
$$\left(\gamma^1\right)^2 = \left(\gamma^2\right)^2 = \left(\gamma^3\right)^2 = -1$$

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 0 \quad (\mu \neq \nu)$$

$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$ ならば $\left(\gamma^k p_k - mc\right)\left(\gamma^\lambda p_\lambda + mc\right) = p^\mu p_\mu - (mc)^2$ ・・・①

$p^\mu p_\mu = \left(\frac{E}{c}\right)^2 - {p_x}^2 - {p_y}^2 - {p_z}^2 = \left(\frac{E}{c}\right)^2 - p^2 =$ 一定(ローレンツスカラー量）・・・②

$p^\mu = \left(mc \quad 0 \quad 0 \quad 0\right)$ の場合は、静止しているときなので

$p^\mu p_\mu = \left(\frac{E}{c}\right)^2 - p^2 = (mc)^2$ $\quad\therefore\quad p^\mu p_\mu - (mc)^2 = 0$ ・・・③

となる。

$\therefore\quad \left(\gamma^k p_k - mc\right)\left(\gamma^\lambda p_\lambda + mc\right) = 0$ ・・・④

従って $\gamma^k p_k - mc = 0$ または $\gamma^\lambda p_\lambda + mc = 0$ が成立する。

$p_\mu = i\hbar\partial_\mu$ なので

$\gamma^\mu i\hbar\partial_\mu - mc = 0$ $\quad \left(i\hbar\gamma^\mu\partial_\mu - mc\right)\psi = 0$ ・・・⑤

$\hbar = c = 1$ とおけば

$\left(i\gamma^\mu\partial_\mu - m\right)\psi = 0$ …⑥ となる。Dirac equation