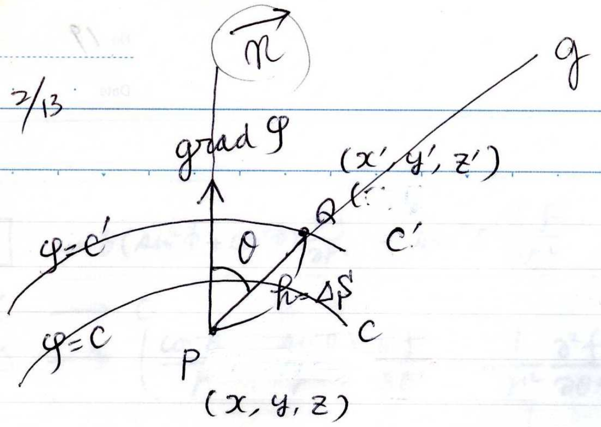
No. 20
Date 2/13



$\Delta\varphi = C' - C$
(増加分)

曲面Cは $\varphi(x, y, z) = C$ を満たす
曲面C'は $\varphi(x', y', z') = C'$ を満たすとする.

P→Qに移動するとき $\varphi(x, y, z)$ の増加は
$x' - x = dx$　$y' - y = dy$　$z' - z = dz$ とすると
$\cancel{df} =$

全微分より

$$d\varphi = \frac{\partial \varphi}{\partial x}dx + \frac{\partial \varphi}{\partial y}dy + \frac{\partial \varphi}{\partial z}dz \quad \text{—①}$$

$$\begin{cases} \mathrm{grad}\,\varphi = \dfrac{\partial \varphi}{\partial x}\vec{i} + \dfrac{\partial \varphi}{\partial y}\vec{j} + \dfrac{\partial \varphi}{\partial z}\vec{k} \\ d\vec{r} = dx\cdot\vec{i} + dy\cdot\vec{j} + dz\cdot\vec{k} \end{cases}$$ とおくと

$$d\varphi = \mathrm{grad}\,\varphi \cdot d\vec{r} = \nabla\varphi \cdot d\vec{r}$$
$$= |\mathrm{grad}\,\varphi|\cdot|d\vec{r}|\cos\theta \quad \text{—②} = |\nabla\varphi|\cdot|d\vec{r}|\cos\theta$$

ここで grad φ と $\overrightarrow{PQ}$ の向きが一致するとき
$\theta = 0$

$$\therefore\ d\varphi = |\mathrm{grad}\,\varphi|\cdot|d\vec{r}|\cos 0$$
$$= |\mathrm{grad}\,\varphi||\cdot d\vec{r}| \quad \text{—③}$$
$$= |\nabla\varphi|\cdot|d\vec{r}|$$

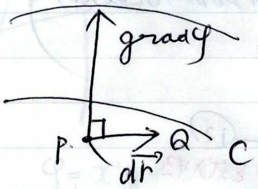
grad φ の向きが $d\varphi$ の増加の最大 の向きである。

また $\overrightarrow{PQ}$ が曲面C上にあるときは $\varphi = C$ (一定) なので

$$d\varphi = \text{grad}\,\varphi \cdot d\vec{r} = 0 \quad \text{—④} \quad (\text{全微分})$$

important

grad φ は曲面Cに対して垂直であり、φ の増加の最大の向きである



$\overline{PQ} = \Delta S$ とおくと

$\overrightarrow{PQ}$ 方向の方向微係数は

$$\frac{\Delta\varphi}{\Delta S} = \frac{d\varphi}{ds} = \frac{\partial\varphi}{\partial x}\frac{dx}{ds} + \frac{\partial\varphi}{\partial y}\frac{dy}{ds} + \frac{\partial\varphi}{\partial z}\cdot\frac{dz}{ds} \quad \text{—⑤}$$

$$= \left(\frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds}\right) = (l, m, n) \quad \text{—⑥}$$

$\overrightarrow{PQ}$ の方向の 単位ベクトル $\vec{u} = (l, m, n)$ —⑦

$$\frac{\Delta\varphi}{\Delta S} = \frac{\partial\varphi}{\partial x}l + \frac{\partial\varphi}{\partial y}\cdot m \cdot \frac{\partial\varphi}{\partial z}\cdot n$$

$$= \vec{u}\cdot \text{grad}\,\varphi \quad \left(\nabla\varphi = \frac{\partial\varphi}{\partial x}\vec{i} + \frac{\partial\varphi}{\partial y}\vec{j} + \frac{\partial\varphi}{\partial z}\vec{k}\right)$$

$$= \vec{u}\cdot\nabla\varphi \quad \text{—⑧}$$

$$= \left(l\cdot\frac{\partial}{\partial x} + m\frac{\partial}{\partial y} + n\frac{\partial}{\partial z}\right)\varphi \quad \text{—⑨}$$

$$\boxed{\frac{\Delta\varphi}{\Delta S} = (\vec{u}\cdot\nabla)\varphi \quad \text{—⑩}}$$

→ $\vec{u}$ 方向の方向微係数

曲面に対して垂直な法線ベクトル $\vec{n}$

$$\boxed{\vec{n} = \frac{\nabla\varphi}{|\nabla\varphi|} \quad \text{—⑪}}$$

(この法線ベクトルは $\varphi = C$ に垂直であり、φの増加する最大の向きである)

$$\frac{\Delta\varphi}{\Delta S} = \mathrm{grad}\,\varphi \cdot \vec{u} \quad \text{—⑫}$$

法線ベクトル⑪を代入する

法線方向の微係数

$$\frac{d\varphi}{dn} = \mathrm{grad}\,\varphi \cdot \vec{n} \quad \text{—⑬}$$

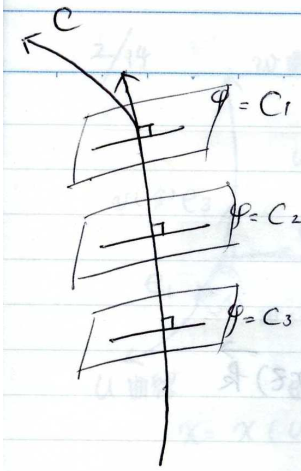
$$\boxed{\frac{\partial\varphi}{\partial n} = \nabla\varphi \cdot \frac{\nabla\varphi}{|\nabla\varphi|} = \frac{|\nabla\varphi| \cdot |\nabla\varphi| \cos 0}{|\nabla\varphi|} = |\nabla\varphi| = |\mathrm{grad}\,\varphi|}$$

⑪より

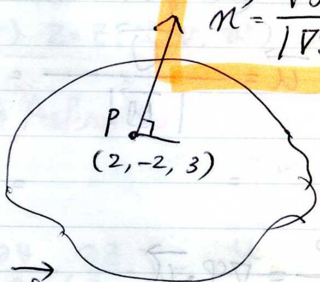
$$\therefore \frac{\partial\varphi}{\partial n} = |\mathrm{grad}\,\varphi| \quad \text{—⑭} > 0 \text{ (増加の向き)}$$

⑪より $\therefore \mathrm{grad}\,\varphi = |\mathrm{grad}\,\varphi| \cdot \vec{n}$ 　 $\nabla\varphi = |\nabla\varphi| \cdot \vec{n}$

$$\boxed{\begin{bmatrix} \nabla\varphi = \dfrac{\partial\varphi}{\partial n} \cdot \vec{n} \quad \text{—⑮} \\ \mathrm{grad}\,\varphi = \dfrac{\partial\varphi}{\partial n} \cdot \vec{n} \quad \text{—⑯} \end{bmatrix}}$$



$$\boxed{\vec{n} = \frac{\nabla\varphi}{|\nabla\varphi|}}$$



[12] $\varphi = x^2y + 2xz = 4 \qquad P(2, -2, 3)$

$$\nabla\varphi = \frac{\partial\varphi}{\partial x}\vec{i} + \frac{\partial\varphi}{\partial y}\vec{j} + \frac{\partial\varphi}{\partial z}\vec{k}$$

$$= (2xy + 2z)\vec{i} + (x^2)\vec{j} + 2x\cdot\vec{k}$$

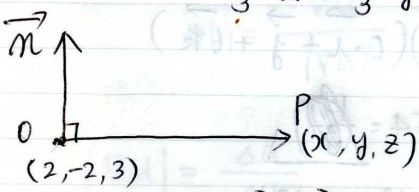
$$= (2\cdot 2\cdot(-2) + 2\cdot 3)\vec{i} + 2^2\vec{j} + 2\cdot 2\vec{k}$$

$$= (-2)\vec{i} + 4\vec{j} + 4\vec{k}$$

$$|\nabla\varphi| = \sqrt{(-2)^2 + 4^2 + 4^2} = \sqrt{4 + 32} = \sqrt{36} = 6$$

$$\vec{n} = \frac{\nabla\varphi}{|\nabla\varphi|} = \frac{-2}{6}\vec{i} + \frac{4}{6}\vec{j} + \frac{4}{6}\vec{k}$$

$$= -\frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{2}{3}\vec{k}$$



$$\vec{OP}\cdot\vec{n} = 0$$

$$\therefore\ (x-2)\left(-\frac{1}{3}\right) + (y-(-2))\frac{2}{3} + (z-3)\frac{2}{3} = 0$$

$$-(x-2) + 2(y+2) + 2(z-3) = 0$$

$$-x + 2 + 2y + 4 + 2z - 6 = 0$$

$$-x + 2y + 2z = 0 \qquad \underline{x - 2y - 2z = 0}$$

[12] $\left[\begin{array}{l} \varphi = x^2yz + 4xz^2 \qquad (1,-2,1) \\ \vec{U} = 2\vec{i} - \vec{j} - 2\vec{k} \text{ への方向微係数を求める} \end{array}\right]$

$$\nabla\varphi = \frac{\partial\varphi}{\partial x} i + \frac{\partial\varphi}{\partial y} j + \frac{\partial\varphi}{\partial z} k$$

$$= (2xyz + 4z^2)\vec{i} + (x^2z)\vec{j} + (x^2y + 8xz)\vec{k}$$

$$\vec{u} = \frac{\vec{U}}{|\vec{U}|} = \frac{2i - j - 2k}{\left|\sqrt{2^2 + (-1)^2 + (-2)^2}\right|}$$

$$= \frac{2i - j - 2k}{\sqrt{9}} = \frac{2}{3}i - \frac{1}{3}j - \frac{2}{3}k$$

$$\frac{\partial\varphi}{\partial u} = \nabla\varphi \cdot \vec{u}$$

$(1,-2,1)$ のとき

$$\nabla\varphi = \{2\cdot1\cdot(-2)\cdot1 + 4\times1^2\}i + (1^2\cdot1)j + (1^2\cdot(-2) + 8\cdot1\cdot1)k$$

$$= (-4+4)i + j + 6k$$

$$= \vec{j} + 6\vec{k}$$

$$\therefore \frac{\partial\varphi}{\partial u} = \left(\frac{2}{3}\vec{i} - \frac{1}{3}\vec{j} - \frac{2}{3}\vec{k}\right)(0\cdot\vec{i} + \vec{j} + 6\vec{k})$$

$$= -\frac{1}{3} - \frac{2}{3}\times 6 = -\frac{1}{3} - 4 =$$

$$= -\frac{1}{3} - \frac{12}{3} = -\frac{13}{3}$$