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DATE 29 · 11 · 22

$p'^{\mu} = L p^{\mu}$

$p'_{\mu} = G L p^{\mu}$

$p'^{\mu} p'_{\mu} = L p^{\mu}$

$\vec{P}' = L\vec{P}$

$\vec{P}^{t} \cdot G\vec{P}$

$\vec{P}' = L\vec{P}$

$\vec{P}'^{t} G \vec{P}' = (L\vec{P})^{t} G (L\vec{P})$

$= \vec{P}^{t} L^{t} G L \vec{P} = \vec{P}^{t} G \vec{P}$

$L = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

L^{t} GL

$\begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ +\beta\gamma & -\gamma & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

G

$= \begin{bmatrix} \gamma^2 + \gamma^2\beta^2 - \beta\gamma^2 + \beta\gamma^2 & 0 & 0 \\ -\beta\gamma^2 + \beta\gamma^2 & \beta^2\gamma^2 - \gamma^2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

G L

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ +\beta\gamma & -\gamma & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

$\begin{cases} \gamma^2 - \beta^2\gamma^2 = (1-\beta^2)\gamma^2 = (1-\beta^2)\dfrac{1}{1-\beta^2} = 1 \\ (\beta^2 - 1)\gamma^2 = -(1-\beta^2)\gamma^2 = -1 \end{cases}$

$L^{t} G L = G$

$\vec{P} = \begin{bmatrix} \frac{E}{c} \\ p^1 \\ p^2 \\ p^3 \end{bmatrix} = \begin{bmatrix} \frac{E}{c} \\ p_x \\ p_y \\ p_z \end{bmatrix}$

$L = L^{t}$

$\begin{bmatrix} \frac{E}{c} & p_x & p_y & p_z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{E}{c} \\ p_x \\ p_y \\ p_z \end{bmatrix}$

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$\vec{P}' = L\vec{P}$

$\vec{P}^{t} G \vec{P}$

$\vec{P}' = L\vec{P}$

$\vec{P}'^{t} G \vec{P}' = (L\vec{P})^{t} G (L\vec{P})$

$= \vec{P}^{t} L^{t} G L \vec{P} = \vec{P}^{t} G \vec{P}$

$\therefore \vec{P}^{t} G \vec{P}$ は Lorentz 変換で 保存される.

ローレンツスカラー量である。