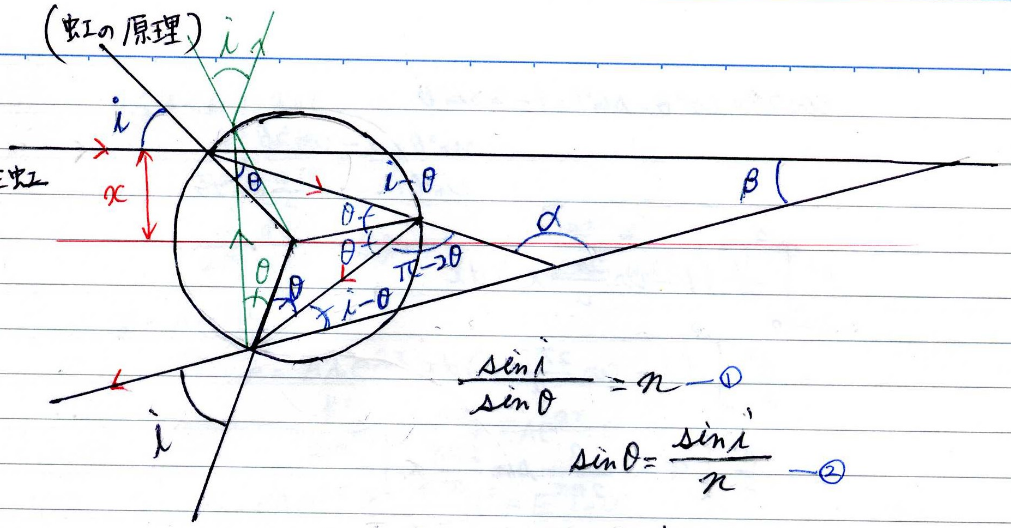
(虹の原理)

① 主虹



$$\frac{\sin i}{\sin\theta} = n \quad ①$$

$$\sin\theta = \frac{\sin i}{n} \quad ②$$

$$\alpha = i-\theta + \pi - 2\theta = \pi + i - 3\theta$$

$$\beta = \pi - (i-\theta) - (\pi + i - 3\theta)$$

$$\beta = -i+\theta-i+3\theta = 4\theta - 2i = 4\sin^{-1}\left(\frac{\sin i}{n}\right) - 2i \quad ③$$

$$\frac{d\beta}{di} = 4\frac{d\theta}{di} - 2 = 4\left(\frac{d\theta}{di} - \frac{1}{2}\right) \quad ④$$

②の関係式より

$$\sin\theta - \frac{\sin i}{n} = 0 \quad ⑤$$

$$\cos\theta\,\frac{d\theta}{di} - \frac{1}{n}\cos i = 0 \quad ⑥$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{\sin i}{n}\right)^2} = \frac{\sqrt{n^2-\sin^2 i}}{n}$$

$$\frac{d\theta}{di} = \frac{1}{n}\,\frac{\cos i}{\cos\theta} \quad ⑦$$

$$\frac{1}{n}\,\frac{\cos i}{\frac{\sqrt{n^2-\sin^2 i}}{n}} = \frac{\cos i}{\sqrt{n^2-\sin^2 i}} \quad ⑧$$

$$\frac{d\beta}{di} = 4\left(\frac{\cos i}{\sqrt{n^2-\sin^2 i}} - \frac{1}{2}\right) \quad ⑨$$

$$\frac{d\beta}{di} = 0 \quad ⑩$$

$$\frac{\cos i_0}{\sqrt{n^2-\sin^2 i_0}} = \frac{1}{2} \quad ⑪$$

$$\frac{\cos^2 i_0}{n^2-\sin^2 i_0} = \frac{1}{4}$$

$$4\cos^2 i_0 = n^2 - \sin^2 i_0$$

$$4(1-\sin^2 i_0) = n^2 - \sin^2 i_0$$

$$4 - 4\sin^2 i_0 = n^2 - \sin^2 i_0$$

$$4 - n^2 = 3\sin^2 i_0 \qquad \sin^2 i_0 = \frac{4-n^2}{3}$$

$$\sin i_0 = \sqrt{\frac{4-n^2}{3}} \quad ⑫$$

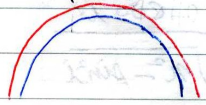
$$\beta = \left\{4\sin^{-1}\left(\frac{\sqrt{\frac{4-n^2}{3}}}{n}\right) - 2\sin^{-1}\left(\sqrt{\frac{4-n^2}{3}}\right)\right\} \times \frac{180^\circ}{\pi}$$

$n=1.333$のとき $\beta = 42.0781$ と計算される

βはθが小さい程小さくなる → 紫色・青色の角が小さい

赤 〃 大きい

(主虹)



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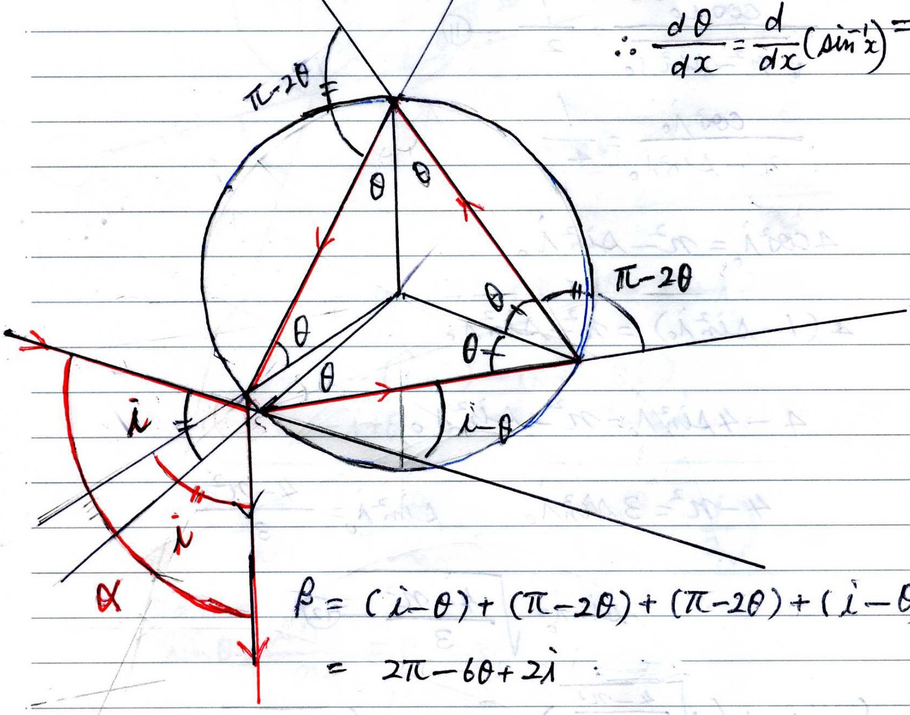
(副虹)

$y = \sin\theta \qquad x = \sin\theta$

$\theta = \sin^{-1}x$

$\theta = \sin^{-1}y \qquad \dfrac{d\theta}{dx} = \dfrac{1}{\dfrac{dx}{d\theta}} = \dfrac{1}{\cos\theta} = \dfrac{1}{\sqrt{1-\sin^2\theta}}$

$\therefore \dfrac{d\theta}{dx} = \dfrac{d}{dx}(\sin^{-1}x) = \dfrac{1}{\sqrt{1-x^2}}$



$\beta = (i-\theta)+(\pi-2\theta)+(\pi-2\theta)+(i-\theta)$

$= 2\pi - 6\theta + 2i$

$\alpha = \beta - \pi = (2\pi - 6\theta + 2i) - \pi = \pi - 6\theta + 2i$

$\alpha = 2i - \underbrace{6\theta}_{+\pi} = 2i - 6\sin^{-1}\left(\dfrac{\sin i}{n}\right) + \pi$

$\dfrac{d\alpha}{di} = 2 - 6\dfrac{d\theta}{di} = 0$

$2 = 6\dfrac{d\theta}{di} \qquad 1 = 3\dfrac{\cos i}{\sqrt{n^2 - \sin^2 i}}$

$$\frac{1}{3}=\frac{\cos i_0}{\sqrt{n^2-\sin^2 i_0}}$$

$$\frac{1}{9}=\frac{\cos^2 i_0}{n^2-\sin^2 i_0} \rightarrow n^2-\sin^2 i_0 = 9\cos^2 i_0{}^2$$

$$n^2-\sin^2 i_0{}^2 = 9(1-\sin^2 i_0)$$

$$n^2-\sin^2 i_0{}^2 = 9-9\sin^2 i_0$$

$$8\sin^2 i_0 = 9-n^2$$

$$\sin^2 i_0=\frac{9-n^2}{8} \qquad \sin i_0=\frac{\sqrt{9-n^2}}{2\sqrt{2}}$$

$$\sin i_0=\sqrt{\frac{9-n^2}{8}}$$

$$\alpha = 2\left(i-3\sin^{-1}\left(\frac{\sqrt{\frac{9-n^2}{8}}}{n}\right)\right)+\pi$$

$$\alpha = 2\left\{\sin^{-1}\sqrt{\frac{9-n^2}{8}}-3\sin^{-1}\frac{\sqrt{\frac{9-n^2}{8}}}{n}\right\}\times\frac{180^\circ}{\pi}+180^\circ$$

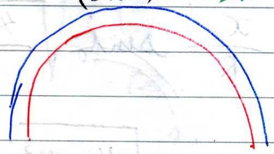
$n=1.333$の時

$\alpha=50.89^\circ$

θが小さい程 αは大きい → 紫色(青)の角が大きい

→ 赤 小さい

(副虹) → 主虹と逆関係となる



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$$x = \cos y$$

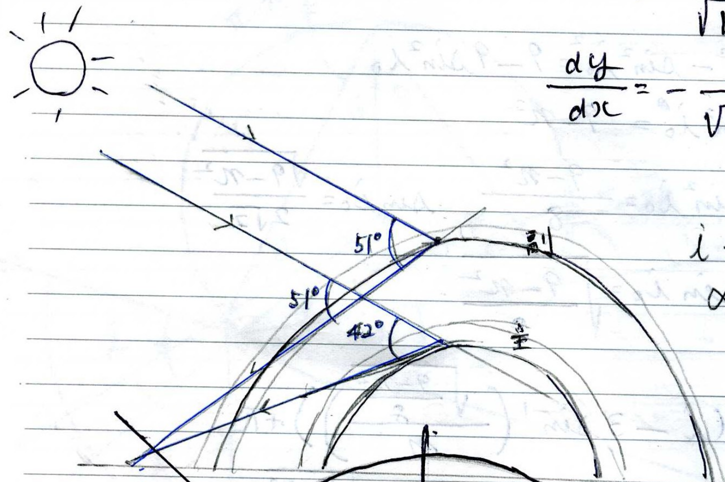
$\theta =$

$$y = \cos^{-1} x \qquad \frac{dx}{dy} = -\sin y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = -\frac{1}{\sin y} \quad \cancel{= -\frac{1}{\sqrt{1-\cos^2 y}}}$$

$$= -\frac{1}{\sqrt{1-\cos^2 y}} \quad \cancel{= -\frac{1}{\sqrt{1-x^2}}}$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$



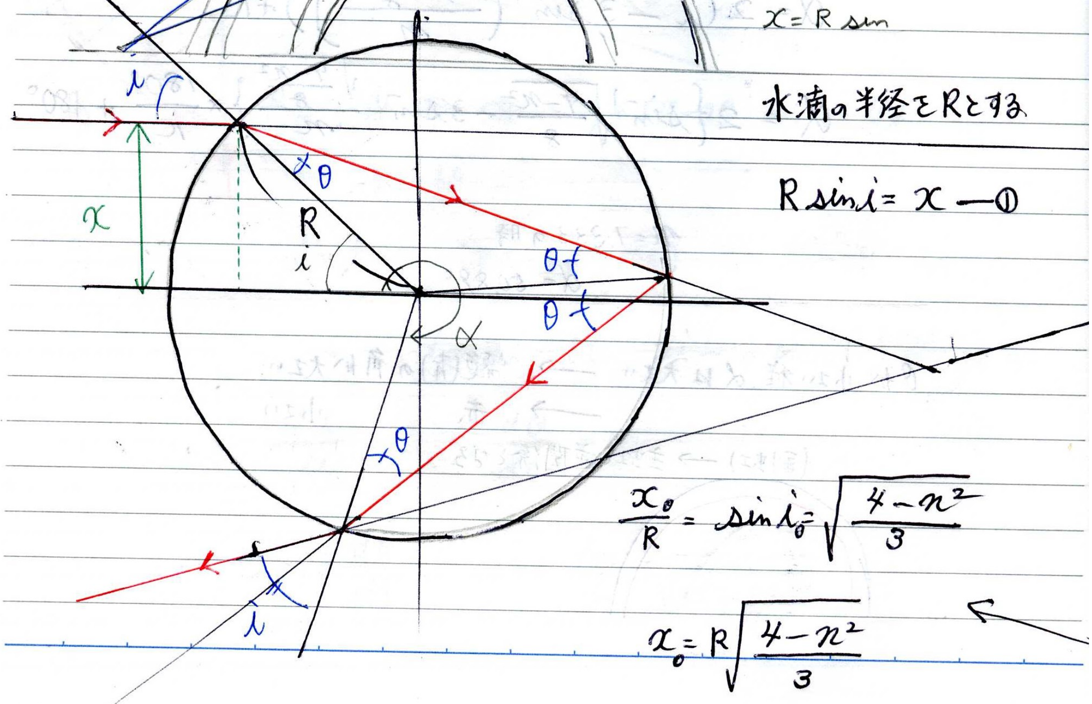
$$i + (\pi - 2\theta) + (\pi - 2\theta)$$

$$\alpha = i + 2\pi - 4\theta$$

$$\alpha' = i + \pi - 4\theta$$

$$\alpha' = 4\theta - i - \pi$$

$$x = R \sin$$



水滴の半径をRとする

$$R \sin i = x \quad —①$$

$$\frac{x_0}{R} = \sin i_0 = \sqrt{\frac{4-n^2}{3}}$$

$$x_0 = R\sqrt{\frac{4-n^2}{3}}$$

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$$\alpha = 回転角 = (i-\theta)+(\pi-2\theta)+(i-\theta)$$

$$\alpha = 2i-4\theta+\pi \quad —②$$

$$\therefore\ \beta = \pi-\alpha = \pi-(2i-4\theta+\pi) = 4\theta-2i \quad —③$$

$$\frac{\sin i}{\sin\theta} = n \quad —④$$

$$\therefore\ \sin\theta = \frac{\sin i}{n} \qquad \theta = \sin^{-1}\left(\frac{\sin i}{n}\right) \quad —⑤$$

$$\therefore\ \beta = 4\sin^{-1}\left(\frac{\sin i}{n}\right) - 2i \quad —⑥$$

① より $\sin i = \dfrac{x}{R}$ とおける

③ を x で微分する

$$\frac{d\beta}{dx} = \frac{d\beta}{di}\times\frac{di}{dx}$$

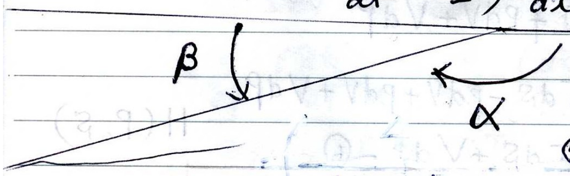
$$= 4\frac{d\theta}{di}\cdot\frac{di}{dx} - 2\frac{di}{dx}$$

$$= 4\left(\frac{d\theta}{di}-\frac{1}{2}\right)\frac{di}{dx} \quad —⑦$$

$$i = \sin^{-1}\frac{x}{R}$$

$$\frac{di}{dx} = \frac{1}{\sqrt{1-\left(\frac{x}{R}\right)^2}}\times\frac{1}{R}$$

$$\frac{di}{dx} = \frac{1}{\sqrt{R^2-x^2}} > 0$$



$\dfrac{d\beta}{dx} = 0$ のとき $\dfrac{di}{dx} > 0$ なので

⑦ より $\dfrac{d\theta}{di} - \dfrac{1}{2} = 0$

$$\frac{d\theta}{di} = \frac{1}{2}$$

$$\frac{1}{\sqrt{1-\left(\frac{\sin i}{n}\right)^2}}\times\frac{1}{n}\times\cos i = \frac{\cos i}{\sqrt{n^2-\sin^2 i}} = \frac{1}{2}$$

$$\cos^2 i = \frac{1}{4}(n^2-\sin^2 i) \qquad \frac{3}{4}\sin^2 i = \frac{4-n^2}{4}$$

$$1-\sin^2 i = \frac{n^2}{4} - \frac{1}{4}\sin^2 i \qquad \sin^2 i = \frac{4-n^2}{3}$$