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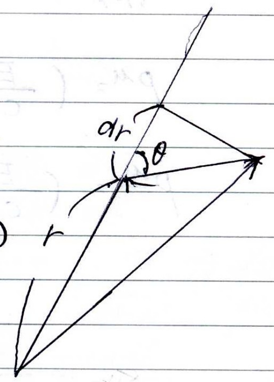
（特殊相対論的効果による近日点の移動）

$$\vec{F} = -\frac{k}{r^2}\frac{\vec{r}}{r} \quad \text{—①}$$

$$\vec{F}\cdot\vec{v} = \frac{d}{dt}\left(\frac{m_0 c^2}{\sqrt{1-\beta^2}}\right) \quad \text{—②}$$



$$\frac{d}{dt}\left(\frac{m_0}{\sqrt{1-\beta^2}}\vec{v}\right) = \vec{F} \quad \text{—③}$$

$$-\frac{k}{r^2}\frac{\vec{r}}{r}\cdot\vec{v} = \frac{d}{dt}\left(\frac{m_0 c^2}{\sqrt{1-\beta^2}}\right) \quad \text{—④}$$

$$-\int \frac{k}{r^3}\vec{r}\cdot\frac{d\vec{r}}{dt}\,dt = \frac{m_0 c^2}{\sqrt{1-\beta^2}} \quad \text{—⑤}$$

$$\left[\begin{aligned} &-\int \frac{k}{r^3}\vec{r}\cdot d\vec{r} = \frac{k}{r} + E \\ &-\int \frac{k}{r^3} r\,dr = -\int \frac{k}{r^2}dr = -k\left[-\frac{1}{r}\right] = k\left[\frac{1}{r} - \frac{1}{r_0}\right] \end{aligned}\right]$$

$$\frac{m_0 c^2}{\sqrt{1-\beta^2}} = \frac{k}{r} + E \quad \text{—⑥}$$

$$\frac{m_0 c^2}{\sqrt{1-\beta^2}} - \frac{k}{r} = E \quad \text{—⑦}$$

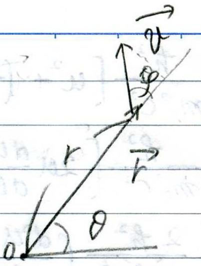
$$\frac{m_0 c^2}{\sqrt{1-\beta^2}} = E + \frac{k}{r} \qquad \frac{m_0^2 c^4}{1-\beta^2} = \left(E + \frac{k}{r}\right)^2$$

$$\frac{1}{1-\beta^2} = \left(\frac{E + \frac{k}{r}}{m_0 c^2}\right)^2 \quad \text{—⑧}$$

$$\frac{d}{dt}(m\vec{v})\times\vec{r} = \vec{F}\times\vec{r}$$

$$= -\frac{k}{r^2}\frac{\vec{r}}{r}\times\vec{r} = 0 \quad \text{—⑨}$$

$$\frac{d}{dt}(m\vec{v}\times\vec{r}) = \frac{d}{dt}(m\vec{v})\times\vec{r} + m\vec{v}\times\frac{d\vec{r}}{dt} = \vec{0}$$



$$\frac{d}{dt}(m\vec{v}\times\vec{r}) = \vec{0} \quad \text{—⑩}$$

$$\therefore \; m\vec{v}\times\vec{r} = \vec{h} \quad \text{—⑪}$$

$$h = mvr\sin\varphi$$

$$= mrv\sin\varphi = mr\cdot r\dot{\theta} = mr^2\dot{\theta} = h\,(\text{一定}) \quad \text{—⑫}$$

$$u = \frac{1}{r} \qquad r = \frac{1}{u}$$

$$r^2\dot{\theta}^2 = r^2\left(\frac{h}{mr^2}\right)^2 = r^2\frac{h^2}{m^2r^4} = \frac{h^2}{mr^2} = \frac{h^2}{m}u^2 \quad \text{—⑬}$$

$$\dot{r}^2 = \left(\frac{dr}{dt}\right)^2 = \left(\frac{dr}{du}\cdot\frac{du}{d\theta}\cdot\frac{d\theta}{dt}\right)^2 \quad \text{—⑭}$$

$$= \left(-\frac{1}{u^2}\times\frac{du}{d\theta}\right)^2\left(\frac{d\theta}{dt}\right)^2 = \frac{1}{u^4}\times\left(\frac{du}{d\theta}\right)^2\times\frac{h^2}{m}u^2\times u^2$$

$$\dot{r}^2 = \frac{h^2}{m}\left(\frac{du}{d\theta}\right)^2 \quad \text{—⑮}$$

$$\beta^2 = \frac{v^2}{c^2} = \frac{1}{c^2}(r^2\dot{\theta}^2 + \dot{r}^2) = \frac{1}{c^2}\left\{\frac{h^2}{m}u^2 + \frac{h^2}{m}\left(\frac{du}{d\theta}\right)^2\right\}$$

$$= \frac{h^2}{mc^2}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] \quad \text{—⑯}$$

$$\beta^2 = \frac{h^2}{c^2}\left(\frac{\sqrt{1-\beta^2}}{m_0}\right)^2\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right]$$

$$\beta^2 = \frac{h^2}{c^2}\cdot\frac{(1-\beta^2)}{m_0^2}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] \quad \text{—⑰}$$

$$\frac{\beta^2}{1-\beta^2} = \frac{h^2}{m_0^2c^2}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] \quad \text{—⑱}$$

$$\frac{1}{1-\beta^2} = \frac{\beta^2}{1-\beta^2} + 1 = \frac{h^2}{m_0^2c^2}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] + 1$$

$$\frac{h^2}{m_0^2c^2}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] + 1 = \left(\frac{E+ku}{m_0c^2}\right)^2 \quad \text{—⑲}$$

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$$\frac{h^2}{m_0^2C^2}\left[u^2+\left(\frac{du}{d\theta}\right)^2\right]+1=\frac{(E+ku)^2}{m_0^2C^4} \quad -⑭$$

$$\frac{h^2}{m_0^2C^2}\left[\cancel{2}u\frac{du}{d\theta}+\cancel{2}\left(\frac{du}{d\theta}\right)\left(\frac{d^2u}{d\theta^2}\right)\right]=\frac{1}{m_0^2C^4}\times\cancel{2}(E+ku)\ k\frac{du}{d\theta}$$

$$\cancel{\frac{du}{d\theta}}\ \frac{\cancel{2}h^2}{\cancel{m_0^2}C^2}\left(\frac{d^2u}{d\theta^2}+u\right)=\frac{\cancel{2}}{\cancel{m_0^2}C^{\cancel{4}\,2}}(E+ku)k\cancel{\frac{du}{d\theta}}$$

$$\frac{d^2u}{d\theta^2}+u=\frac{1}{h^2C^2}(Ek+k^2u)$$

$$\frac{d^2u}{d\theta^2}+\left[\left(1-\frac{k^2}{h^2C^2}\right)\right]u=\frac{Ek}{h^2C^2} \quad -⑮$$

$$\left[\gamma^2=1-\frac{k^2}{h^2C^2}\right]$$

$$\frac{d^2u}{d\theta^2}+\gamma^2(u-u_0)=0 \quad -⑯$$

$$\left[-\gamma^2u_0=-\frac{Ek}{h^2C^2} \qquad u_0=\frac{Ek}{\gamma^2h^2C^2}\right]$$

一般解として $u-u_0=A\cos(\gamma\theta+\alpha)$ —⑰ とおくと

$$\frac{du}{d\theta}=-A\gamma\sin(\gamma\theta+\alpha)$$

$$\frac{d^2u}{d\theta^2}=-A\gamma^2\cos(\gamma\theta+\alpha)$$

$$u=\frac{1}{r}=u_0+A\cos(\gamma\theta+\alpha)$$

$$\boxed{\begin{aligned}&\frac{d^2u}{d\theta^2}+\gamma^2(u-u_0)\\&=-A\gamma^2\cos(\gamma\theta+\alpha)+A\gamma^2\cos(\gamma\theta+\alpha)\\&=0\end{aligned}}$$

$$r=\frac{1}{u_0+A\cos(\gamma\theta+\alpha)}=\frac{\frac{1}{u_0}}{1+\frac{A}{u_0}\cos(\gamma\theta+\alpha)} \quad -⑱$$

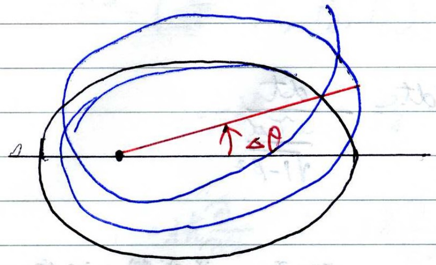
$$\varepsilon=\frac{A}{u_0} \quad l=\frac{1}{u_0} \qquad r=\frac{l}{1+\varepsilon\cos(\gamma\theta+\alpha)} \quad -⑲$$

近日点の条件は

$$\gamma\theta+\alpha=2\pi n \quad -⑳ \qquad \Delta\theta_n-2\pi$$

$$\theta_n=\frac{2\pi}{\gamma}n-\frac{\alpha}{\gamma} \quad -㉑ \qquad \Delta\theta'=\frac{2\pi}{\gamma}-2\pi=2\pi\left(\frac{1}{\gamma}-1\right)$$

$$\Delta\theta_n=\frac{2\pi}{\gamma} \quad -㉒ \quad >2\pi$$ なので



100光年　　　ロケットで20年かかる

$$\frac{100c}{v}=\frac{20}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$\left(\frac{c}{v}\right)^2\times10^4=\frac{400}{1-\left(\frac{v}{c}\right)^2}$$

$$\left(1-\frac{v^2}{c^2}\right)\times\frac{c^2}{v^2}\times10^4=4\times10^2$$

$$\left(\frac{c^2}{v^2}-1\right)\times10^4=4\times10^2$$

$$\frac{c^2}{v^2}-1=4\times10^{-2}$$

$$\frac{c^2}{v^2}=1.04$$

$$\left(\frac{v}{c}\right)^2=\frac{1}{1.04}=\frac{100}{104}$$

$$\frac{v}{c}=\sqrt{\frac{1}{1.04}}=\sqrt{\frac{100}{104}}$$

muonの固有時 $d\tau=\sqrt{1-\beta^2}dt$ はどの慣性系でも一定

muonが静止して見える慣性系においては

$v'=0$ $\therefore dt'=d\tau$ —①

$d\tau=dt'=\sqrt{1-\beta^2}dt$ —②　　$dt\to$ muonが v で動いて見える慣性系での時間間隔

$$dt=\frac{dt'}{\sqrt{1-\beta^2}}=\gamma dt' \qquad \gamma=\frac{1}{\sqrt{1-\beta^2}}>1$$

$\therefore$ 半減期の時間が γ 倍となる